Long-range magnetic response of the *XY* spin chain under far-from-equilibrium conditions

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Within the formalism of the Keldysh Green's functions we investigate long-range response of an anisotropic *XY* chain to the local magnetic field. This field couples to a single spin on a selected lattice site. The system is driven out of equilibrium by a coupling to two semi-infinite *XX* spin chains. We demonstrate that the long-range response becomes enhanced by a few orders of magnitude upon application of nonequilibrium conditions. This enhancement does not occur in the isotropic *XX* chain. Our results agree with the recently predicted nonequilibrium-driven long-range magnetic correlations [T. Prosen and I. Pižorn, Phys. Rev. Lett. **101**, 105701 (2008)]. We argue that this effect may be observed in quasi-one-dimensional triplet superconductors.

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I. INTRODUCTION

Physical properties of open quantum systems under nonequilibrium conditions remain in many aspects an unexplored field that has recently attracted significant interest. The main reason for such a tendency is a possible application of nanoscopic systems, especially that the process of miniaturization allows for fabrication of conceptually and functionally new devices which operate in the quantum regime. However, the physics of quantum systems driven out of equilibrium raises also novel fundamental questions. One of them concerns the quantum phase transition under nonequilibrium conditions. This problem has recently been analyzed in the context of strongly nonlinear transport in the vicinity of superconductor-insulator[1,2] transition as well as in connection with nonequilibrium magnetic transitions.[3–5] The influence of nonequilibrium conditions upon the basic features characterizing the phase transition is still far from being understood. Investigations of the ferromagnetic phase transition in one-dimensional (1D) system of itinerant electrons have shown that the bias voltage may change its universality class from the uniaxial ferroelectric to the mean-field one.[4] The results of Ref. 3 indicate that the nonequilibrium-induced phase transitions have the same critical behavior as conventional phase transitions, whereas the nonequilibrium drive is responsible mainly for generation of an effective temperature. Nonequilibrium conditions have also been considered as a driving mechanism for metal-insulator[6] and metal-superconductor transitions.[7] Quantum phase transition under nonequilibrium conditions has also been studied in the context of the impurity problem.[8,9]

The recently reported studies of the *XY* spin chain have provided arguments for a nonequilibrium quantum phase transition in this system. It shows up as onset of the *long-range magnetic correlations* (LRMC).[5,10] These investigations have been carried out within the setup[11] which allows for an exact solution of the many-body quantum master equation.[12] Within this setup, the bath operators (expressed in terms of the spin-ladder operators) act on the first and last spin of the *XY* chain. It has been shown that LRMC occur under nonequilibrium conditions only in anisotropic *XY* chain and vanish in the limiting case of the *XX* model. As shown in Ref. 13, the steady state of the *XX* chain is characterized by a flat magnetization profile also in a contact with repeated-interaction reservoirs. A direct method for solving the problem of the long-range order is to show that the spin-spin correlator $\langle S_i S_j\rangle - \langle S_j\rangle\langle S_i\rangle$ does not vanish for $|R_i - R_j| \to \infty$. Unfortunately, as pointed out in Ref. 5, experimental verification of this theoretical prediction may be difficult due to very small values of the residual spin-spin correlator.

In this work we apply a different method of reasoning that, on the one hand, overcomes the problem of small spin-spin correlator while, on the other hand, it gives qualitative information on the nonequilibrium properties of the *XY* chain. We study the response of the *XY* chain to a *local* magnetic field h_i that couples only to spin on site i. If $\langle S_i S_j\rangle - \langle S_j\rangle\langle S_i\rangle$ is nonvanishing for arbitrary $|R_i - R_j|$ then h_i should pin the magnetic structure and, in this way, should modify $\langle S_j\rangle$ at arbitrarily distant site j. A similar argumentation has recently been applied in investigations of an incommensurate charge-density wave state in the presence of a strong impurity potential.[14] If, however, there are no LRMC in the homogeneous *XY* chain, modification of $\langle S_j\rangle$ due to the local magnetic field should decay algebraically with the distance from site i. The latter case should be similar to the Friedel oscillations,[15] which are known to decay with the distance from the impurity as $|r|^{-d}$ in a d-dimensional system. However, the exponent d may be substantially modified by the presence of electronic correlations.[16–22] In the system of noninteracting electrons, a finite bias voltage has been shown to modify the wavelength of the Friedel oscillations, however, the amplitude and the spatial decay exponent of the oscillations remain intact.[23] In the present work we investigate an *XY* system that is coupled to two semi-infinite reservoirs modeled by *XX* spin chains with large exchange interactions. We investigate two scenarios for the nonequilibrium conditions: the driving force originates either from different temperatures or different magnetic fields applied to the reservoirs. Similarly to Refs. 5 and 10 we carry out calculations for large but finite systems and, consequently, we cannot study the response at infinitely distant sites. Nevertheless, our results clearly show that the long-range response of anisotropic *XY* chain is significantly enhanced due to application of nonequilibrium conditions. This enhancement does not take place in the isotropic case, i.e., in the case of the *XX* model. Our results provide strong although indirect support for the hypothesis of the far-from-equilibrium magnetic order in the *XY* chain.

II. MODEL AND APPROACH

We investigate a 1D *XY* chain under nonequilibrium conditions, which are imposed by couplings between the chain and two different macroscopic reservoirs. These reservoirs are modeled by 1D *isotropic XX* chains with *large* exchange interactions. Assumption of the isotropic case is essential, since the energy spectrum of an anisotropic *XY* model may be gaped. Hamiltonian of the system under consideration reads

$$H = H_{XY} + H_L + H_R + H_{\mathrm{coupling}}, \tag{1}$$

where H_{XY} is the Hamiltonian of the *XY* chain whereas H_L and H_R describe left and right reservoirs, respectively. The reservoirs are modeled as follows:

$$H_a = J_a \sum_{m=1}^{N_a-1} (S^+_{a,m}S^-_{a,m+1} + S^+_{a,m+1}S^-_{a,m}) + h_a \sum_{m=1}^{N_a} S^z_{a,m}, \tag{2}$$

where $a=L,R$ distinguishes between the reservoirs, $S^{\pm,z}_{a,m}$ are the spin operators on site m of the reservoir a, and h_a is the external magnetic field that, in general, can be different for left and right reservoirs. This difference will be considered as one of mechanisms, which can drive the system out of equilibrium. The second obvious type of driving originates from different temperatures of the reservoirs. The investigated *XY* chain consists of N sites and its Hamiltonian is of the form

$$H_{XY} = J \sum_{m=1}^{N-1} [S^+_m S^-_{m+1} + S^-_m S^+_{m+1} + \gamma (S^+_m S^+_{m+1} + S^-_m S^-_{m+1})] + h \sum_{m=1}^{N} S^z_m + h_i S^z_i, \tag{3}$$

where h is the uniform magnetic field and the parameter γ describes the anisotropy. From now on we will use J as the energy unit. In Hamiltonian (3) h_i is the local magnetic field that couples only to the spin operator at site i. In the following we study the long-range response to this field. Finally, the coupling between the *XY* chain and reservoirs is assumed to be of the form

$$H_{\mathrm{coupling}} = g_L(S^+_{L,N_L}S^-_1 + S^-_{L,N_L}S^+_1) + g_R(S^+_{R,1}S^-_N + S^-_{R,1}S^+_N). \tag{4}$$

Figure 1 shows the sketch of the investigated setup.

As the total spin system forms a 1D structure, we proceed in the standard way. We apply the Jordan-Wigner transformation[24,25] and introduce fermionic operators $d_m = \exp[i\pi \Sigma_{j<m}(1/2+S^z_j)]S^-_m$. Then, one gets

$$H_a = J_a \sum_{m=1}^{N_a-1} (d^\dagger_{a,m}d_{a,m+1} + d^\dagger_{a,m+1}d_{a,m}) + h_a \sum_{m=1}^{N_a} d^\dagger_{a,m}d_{a,m} + \mathrm{const}, \tag{5}$$

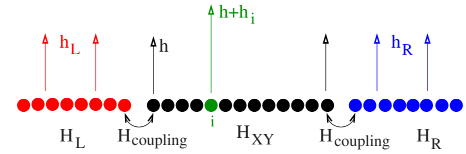

FIG. 1. (Color online) Schematic plot of the setup under investigation. See Eqs. (1)–(4) for the notation.

$$H_{XY} = J \sum_{m=1}^{N-1} [(d^\dagger_m d_{m+1} + d^\dagger_{m+1} d_m) + \gamma (d^\dagger_m d^\dagger_{m+1} + d_{m+1} d_m)] + h \sum_{m=1}^{N} d^\dagger_m d_m + h_i d^\dagger_i d_i + \mathrm{const}, \tag{6}$$

$$H_{\mathrm{coupling}} = g_L(d^\dagger_{L,N_L}d_1 + d^\dagger_1 d_{L,N_L}) + g_R(d^\dagger_{R,1}d_N + d^\dagger_N d_{R,1}) \tag{7}$$

with $a=L,R$ in Eq. (5). A similar setup has recently been investigated in a context of the thermal transport.[26] However, in opposite to our approach, a semi-infinite *XY* chain has been taken as one of the heat reservoirs.

Hamiltonian of the *XY* chain can be diagonalized with the help of the transformation

$$d_m = \sum_n (U_{mn} f_n + V^*_{mn} f^\dagger_n), \tag{8}$$

where f are fermionic operators while the functions U_{mn} and V_{mn} fulfill the Bogoliubov-de Gennes (BdG) equations[27]

$$\sum_{m'} \begin{pmatrix} \mathcal{H}_{mm'} & \Delta_{mm'} \\ -\Delta_{mm'} & -\mathcal{H}_{mm'} \end{pmatrix} \begin{pmatrix} U_{m'n} \\ V_{m'n} \end{pmatrix} = E_n \begin{pmatrix} U_{mn} \\ V_{mn} \end{pmatrix}, \tag{9}$$

where

$$\mathcal{H}_{mm'} = J(\delta_{m',m+1} + \delta_{m',m-1}) + \delta_{mm'}(h + \delta_{mi}h_i), \tag{10}$$

$$\Delta_{mm'} = J\gamma(\delta_{m',m+1} - \delta_{m',m-1}). \tag{11}$$

As the anisotropy parameter γ is fixed, the BdG equations represent $2N \times 2N$ eigenproblem that gives $2N$ eigenvalues E_n and eigenvectors $(U_{1n}\ldots U_{Nn}, V_{1n}\ldots V_{Nn})$. For spinless fermions the BdG equations remain invariant under the transformation

$$U_{mn} \to V_{mn}, \quad V_{mn} \to U_{mn}, \quad E_n \to -E_n. \tag{12}$$

This symmetry allows one to choose N eigenvectors from among $2N$ ones, in such a way that transformation Eq. (8) preserves the fermionic commutation relations. Usually one chooses N solutions with $E_n \le 0$. However, for the reasons that will be explained below, in our approach it is more convenient to choose other combination. The eigenstates obtained from the numerical solution of the BdG equations are grouped as *N pairs* of eigenstates. Within each pair, one state can be obtained from the other by means of the transformation Eq. (12). Then, from each such a pair we choose the state for which $\Sigma_m |U_{mn}|^2 > \Sigma_m |V_{mn}|^2$. For $\gamma \ll 1$ we obtain N states which satisfy $\Sigma_m |U_{mn}|^2 \gg \Sigma_m |V_{mn}|^2$.

The BdG transformation leads to a simple diagonal form of H_{XY}. However, one still needs to transform H_{coupling} according to Eq. (8). Due to the described above choice of eigenstates we neglect V_{mn} in H_{coupling}. This approximation is justified for $\gamma \ll 1$ and it will be tested in the first step of our numerical calculations. Within this approximation we get

$$H_{XY} = \sum_{n=1}^{N} E_n f_n^\dagger f_n, \tag{13}$$

$$H_{\text{coupling}} \simeq g_L \sum_n (U_{1n} d_{L,N_L}^\dagger f_n + U_{1n}^* f_n^\dagger d_{L,N_L}) + g_R \sum_n (U_{Nn} d_{R,1}^\dagger f_n + U_{Nn}^* f_n^\dagger d_{R,1}). \tag{14}$$

This is a standard form of a Hamiltonian, that is used for study of charge transport through noninteracting nanowire. We have applied the formalism of Keldysh Green's functions and calculated $\langle f_n^\dagger f_{n'} \rangle$ in a nonequilibrium steady state. The details are thoroughly described in Refs. 23, 28, and 29. However, for the sake of completeness, we recall the major steps of this approach. After the Jordan-Wigner transformation is applied, the semi-infinite reservoirs are described by one-dimensional tight-binding Hamiltonians of noninteracting fermions [see Eq. (5)]. Due to the geometry of the setup we assume open boundary conditions for H_L and H_R and, for simplicity, we take $N_L = N_R$. Although the latter assumption is not crucial, it allows one to enumerate the eigenstates of the reservoirs with a single set of wave vectors. Then, H_L and H_R can be diagonalized with the help of the unitary transformation[30]

$$d_{a,m}^\dagger = \sqrt{\frac{2}{N_R+1}} \sum_k \sin(km) d_{a,k}^\dagger, \tag{15}$$

where the wave vectors k take on the following values:

$$k = \frac{\pi}{N_R+1}, \frac{2\pi}{N_R+1}, \ldots, \frac{N_R \pi}{N_R+1}. \tag{16}$$

In this representation the reservoirs are described by

$$H_L + H_R = \sum_{a \in \{L,R\}} \sum_k (\varepsilon_{ak} + h_a) d_{a,k}^\dagger d_{a,k} \tag{17}$$

with the dispersion relation $\varepsilon_{ak} = 2J_a \cos(k)$. The coupling between the *XY* chain and the reservoirs can be written as

$$H_{\text{coupling}} = \sum_{a \in \{L,R\}} \sum_{kn} (\tilde{g}_{akn} d_{a,k}^\dagger f_n + \text{H.c.}), \tag{18}$$

where

$$\tilde{g}_{Lkn} = g_L U_{1n} \sqrt{\frac{2}{N_R+1}} \sin(kN_R), \tag{19}$$

$$\tilde{g}_{Rkn} = g_R U_{Nn} \sqrt{\frac{2}{N_R+1}} \sin(k). \tag{20}$$

Since Hamiltonians (13) and (17) describe noninteracting particles, the Keldysh Green's functions can be obtained with the help of the equations of motion. In order to discuss the spatial spin configuration we have calculated

$$\langle f_n^\dagger f_{n'} \rangle = \frac{1}{2\pi i} \int d\omega G_{n',n}^<(\omega). \tag{21}$$

A matrix containing the lesser Green's functions $G_{n',n}^<(\omega)$ can be expressed in terms of the retarded (G^{ret}) and advanced (G^{adv}) Green's functions

$$\hat{G}^<(\omega) = i \sum_{a \in \{L,R\}} \hat{G}^{ret}(\omega) \hat{\Gamma}^a(\omega) \hat{G}^{adv}(\omega) F^a(\omega), \tag{22}$$

where $F^a(\omega)$ denotes the Fermi distribution function $F^a(\omega)^{-1} = \exp[(\omega + h_a)/(k_B T_a)] + 1$ and the retarded Green's function can be calculated from the following formula:

$$\hat{G}^{ret}(\omega) = [\omega \hat{I} - \hat{H} - \hat{\Sigma}^{ret}(\omega)]^{-1}. \tag{23}$$

Here, $\hat{H}$ consists of the matrix elements of H_{XY}, i.e., $H_{nn'} = E_n \delta_{nn'}$ and the retarded self-energy is determined by the spectral functions

$$\hat{\Sigma}^{ret}(\omega) = \frac{1}{2} \sum_{\alpha \in \{L,R\}} \left[\frac{1}{\pi} P \int d\Omega \frac{\hat{\Gamma}^a(\Omega)}{\omega - \Omega} - i \hat{\Gamma}^a(\omega) \right]. \tag{24}$$

Finally, the spectral functions of left and right reservoirs are defined as

$$\Gamma_{nn'}^a(\omega) = [\hat{\Gamma}^a(\omega)]_{nn'} = 2\pi \sum_k \tilde{g}_{akn}^* \tilde{g}_{akn'} \delta(\omega - \varepsilon_{ak}), \tag{25}$$

and for semi-infinite systems ($N_R \to \infty$) one finds

$$\Gamma_{nn'}^L(\omega) = U_{1n}^* U_{1n'} \frac{2g_L^2}{|J_L|} \sqrt{1 - \frac{\omega^2}{4J_L^2}}, \quad |\omega| < 2J_L,$$

$$\Gamma_{nn'}^R(\omega) = U_{Nn}^* U_{Nn'} \frac{2g_R^2}{|J_R|} \sqrt{1 - \frac{\omega^2}{4J_R^2}}, \quad |\omega| < 2J_R.$$

Since we assume $J_a \gg J$, the frequency dependence of the spectral functions can be neglected. A further simplification is possible, when the coupling between the *XY* chain and the reservoirs is symmetric, i.e., $J_L = J_R$, $g_L = g_R$. Then, both the spectral functions can be parametrized by a single quantity Γ_0 as

$$\Gamma_{nn'}^L = U_{1n}^* U_{1n'} \Gamma_0, \tag{26}$$

$$\Gamma_{nn'}^R = U_{Nn}^* U_{Nn'} \Gamma_0. \tag{27}$$

After calculating $\langle f_n^\dagger f_{n'} \rangle$ we have applied the transformation Eq. (8) and determined $\langle S_j^z \rangle = \langle d_j^\dagger d_j \rangle - 1/2$. The results will be discussed in the following section.

III. NUMERICAL RESULTS

We have carried out numerical calculations for a *XY* chain that consists of 172 sites and solved Eqs. (9)–(11) together with Eqs. (21)–(24). The local magnetic field h_i has been applied at site $i = 12$ and we have studied how this field affects $\langle S_j \rangle$ at various lattice sites. However, before presenting the properties of nonequilibrium steady state we have

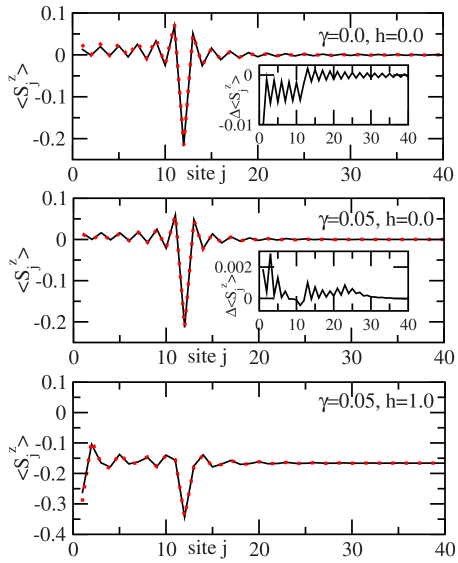

FIG. 2. (Color online) $\langle S_j^z \rangle$ at various lattice sites j obtained under equilibrium conditions with the local field $h_i=1$ applied at site $i=12$. Points show results obtained directly from BdG equations for $k_B T=10^{-5}$. Continuous lines show $\langle S_j^z \rangle$ calculated for the XY chain coupled to two identical reservoirs with $\Gamma_0=0.3$ and $k_B T_L = k_B T_R = 10^{-5}$. The insets in topmost and middle panels show the differences between $\langle S_j^z \rangle$ obtained within both the approaches, $\Delta\langle S_j^z \rangle = \langle S_j^z \rangle_{\text{Keldysh}} - \langle S_j^z \rangle_{\text{BdG}}$. The values of anisotropy γ and the uniform magnetic field h are depicted in the panels.

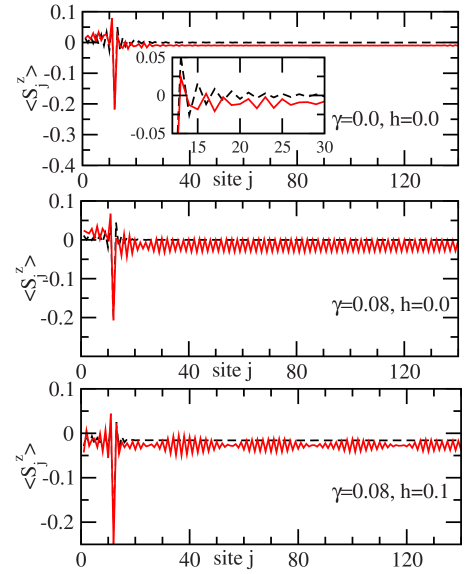

FIG. 3. (Color online) $\langle S_j^z \rangle$ for a 172-site XY chain with the local field $h_i=1$ applied at site $i=12$. Temperatures of the reservoirs are the same $k_B T_L = k_B T_R = 10^{-5}$ and $\Gamma_0=0.3$ has been assumed. Dashed (black) line shows results obtained for the equilibrium limit ($V=0$), whereas the continuous (red) line shows $\langle S_j^z \rangle$ for the driving force $V=1$. The values of γ and h are depicted in the panels. In the inset in the topmost panel we enlarge the region in the vicinity of the site $i=12$.

checked the validity of the approximation Eq. (14). For that purpose, we have carried out calculations within the Keldysh formalism for the equilibrium case when temperatures and all microscopic parameters of left and right reservoirs are equal, i.e., $T_L=T_R$, $h_L=h_R$. Next, we have calculated $\langle S_j^z \rangle$ assuming that the XY chain is in equilibrium with a single thermostat and $\langle f_n^\dagger f_{n'} \rangle$ can be determined directly from the BdG equations as $\langle f_n^\dagger f_{n'} \rangle = \delta_{nn'} F(E_n)$ with $F(x)$ being the Fermi distribution function. In Fig. 2 we show results obtained from these two approaches. On the one hand, the approximation Eq. (14) becomes exact when $\gamma=0$. On the other hand, comparing the insets in the topmost and the middle panels in Fig. 2 one can note that the tiny differences in $\langle S_j^z \rangle$ are independent of γ. This, in turn, suggests that these differences do not originate from the applied approximation. They may occur due to a finite imaginary part of the self-energy that in the Keldysh formalism originates from the coupling to reservoirs. Results shown in Fig. 2 indicate that the approximation Eq. (14) is reasonably accurate provided $\gamma \ll 1$.

The main focus of the present work is to find out whether the long-range magnetic response of the XY chain is affected by the nonequilibrium conditions. These conditions occur due to differences in two reservoirs. In the following we analyze stationary states with the energy flow that originates either from different temperatures of the reservoirs or from the driving force $V=h_R-h_L$ with $h_L=-h_R$. The results obtained for the latter case are shown in Fig. 3. After the Jordan-Wigner transformation has been applied to isotropic XX model ($\gamma=0$), the local magnetic field becomes equivalent to a nonmagnetic impurity placed in a system of noninteracting fermions, whereas the driving force plays the role of the bias voltage. Therefore, h_i gives rise to the (Friedel) oscillations of $\langle S_j^z \rangle$. Under equilibrium conditions these oscillations should vanish as $|R_i-R_j|^{-1}$. One can note that the driving force does not influence the decay of oscillations. As it has been shown in Ref. 23 for noninteracting fermions, the driving force affects the period of oscillations, whereas the envelope of the oscillations is hardly modified. On the contrary, for anisotropic XY chain the driving force substantially modifies the long-range response. While for $V=0$ the oscillations extinguish over the distance of several lattice sites, for a nonzero driving force the oscillations are strong in the whole system. Within the accessible sizes of chain (up to 200 lattice sites) we do not observe any decay of the oscillations.[31] Usually, results obtained for a finite system do not allow one to formulate conclusive statements on the long-range order. However, the nonequilibrium-induced enhancement of the long-range response is appealing. As can be observed in the lowest panel of Fig. 3, the spatial structure of $\langle S_j^z \rangle$ is strongly nonmonotonic in the presence of a magnetic field. Although we are unable to give a simple explanation for these beatlike oscillations, we expect that they may be related to a fine spatial structure of the spin-spin correlation matrices (see Fig. 2 in Ref. 5).

In order to discuss how the long-range response depends on various model parameters, it is necessary to describe it in a quantitative way. As demonstrated above the local magnetic field causes oscillations in the spin distribution and the spatial profiles of these oscillations depends, e.g., on the driving force. Therefore, we have calculated the standard deviation of the spatial spin distribution defined as

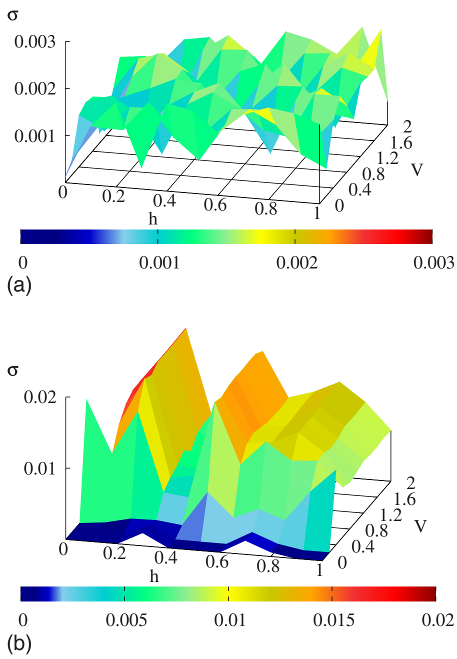

FIG. 4. (Color online) The standard deviation of spatial spin distribution σ [see Eq. (28)] as a function of the driving force V and the magnetic field h for $h_i=1$. The anisotropy parameter is $\gamma=0$ (upper panel) and $\gamma=0.1$ (lower panel). The temperatures of the reservoirs are $k_BT_L=k_BT_R=10^{-5}$ and the coupling constant is $\Gamma_0=0.3$.

$$\sigma = \sqrt{\frac{1}{M+1}\sum_{m=K}^{K+M}(\langle S_m^z\rangle - \bar{S})^2}, \quad \bar{S} = \frac{1}{M+1}\sum_{m=K}^{K+M}\langle S_m^z\rangle. \tag{28}$$

We have carried out numerical calculations for $K=72$ and $M=40$. Within such a choice of the parameters K and M one accounts for oscillations at sites distant both from the site where h_i is applied and from the edge of the chain. In Fig. 4 we show σ as a function of the driving force V and the uniform magnetic field h. For the sake of clarity we additionally present the h cross-sections of these three-dimensional plots (see Fig. 5). One can see that for $\gamma=0$ the magnitude of oscillations is roughly independent of V or h, what indicates that the nonequilibrium conditions have only minor influence on the long-range response of the *XX* chain. However, for $\gamma\neq 0$ finite V may enhance the oscillations by a few orders of magnitude. Particularly strong enhancement occurs when the driving force exceeds a threshold value. Further increase in V above this threshold does not cause any essential modification of the long-range response. We have found that the threshold value of V increases with the anisotropy parameter γ, whereas it decreases when the magnetic field becomes stronger. The threshold value of V and its field dependence can be inferred from the lower panel in Fig. 5. One also notices that for nonzero γ the peaks in σ are insensitive to V (see the lower panel in Fig. 4). This is consistent with the analysis of the *XY* model in terms of the master equation,[5,10] where one sees that the structure of two-point correlator in the bulk is essentially insensitive to the details of the bath,

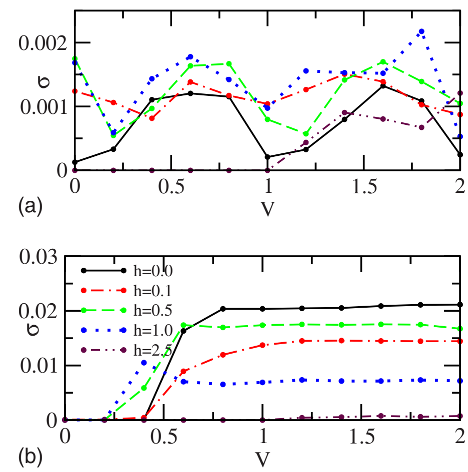

FIG. 5. (Color online) The corresponding h cross-sections of the three-dimensional plots from Fig. 4. The values of magnetic field h are indicated in the legend common for both panels. The points show our numerical results, whereas the lines are guidelines for the eyes.

however, as a function of the bulk parameters, e.g., the uniform magnetic field h, the response of the systems exhibits resonancelike pattern. Indeed it can be shown, that the magnetic response problem in *XY* chain near the critical field $h \approx h_c=2|1-\gamma^2|$ is formally equivalent to a driven Helmholz equation where the magnetic field plays the role of the energy or frequency parameter and the peaks of Fig. 4 may be understood as resonances.[32] Note that in our notation the ratio h/J is twice larger compared to Refs. 5 and 10.

In the middle panel of Fig. 3 nondecaying oscillations are visible even though the magnetic field h is equal to zero, provided the nonequilibrium conditions are imposed. The same conclusion can be drawn on the basis of results presented in the lower panel of Fig. 5. As a consequence one can expect the LRMC to appear in the anisotropic *XY* chain also in the absence of external magnetic field. It is the only important difference between our findings and the results reported in Refs. 5 and 10, where the case $h=0$ is a singular line beyond the boundaries of LRMC. This discrepancy may be due to the fact that in the present work the system contains a nonvanishing magnetic field $h_i\neq 0$, the effect of which might be also smoothing the singularity at $h=0$, in particular, in light of the fact that due to nonergodicity of the *XY* model dynamical properties of the reservoirs can influence dynamical properties of the system. Perhaps such singularity at $h=0$ is then a peculiarity of more restricted models of the reservoirs (as the ones used in master equation approaches[5,10]).

Up to this point, we have discussed results obtained for nonequilibrium conditions induced by V. The driving force originates from different magnetic fields applied to left and right reservoirs. Experimental realization of such conditions may be difficult. They are also very different from the conditions analyzed in Refs. 5 and 10. Therefore, in the following we study the case, when the driving force is zero but the nonequilibrium conditions originate from different temperatures of left and right reservoirs. Figure 6 shows analogous results to those presented in Fig. 3 but for *thermal* driving

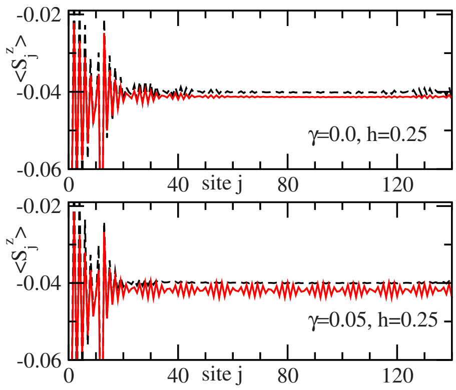

FIG. 6. (Color online) $\langle S_j^z \rangle$ for a 172-site XY chain with the local field h_i=0.25 applied at site i=12. Dashed (black) line shows results obtained for the equilibrium $k_BT_L = k_BT_R = 0.02$, whereas the continuous (red) line shows $\langle S_j^z \rangle$ when driving originates from different temperatures of reservoirs: $k_BT_L = 0.5, k_BT_R = 0.02$ and $\Gamma_0 = 0.3$ has been assumed. The values of γ and h are depicted in the panels.

that originates from different temperatures of the reservoirs. Also in this case one observes a nonequilibrium-driven enhancement of the long-range response of the anisotropic XY model. The field dependence of σ is shown in Fig. 7 for XX (upper panel) and XY (lower panel) chains. Results shown in the upper panel can straightforwardly be explained within the well-known properties of the Friedel oscillations in noninteracting systems. The Friedel oscillations can be understood as the result of interference of incoming and outgoing waves scattered off the impurity. Then, thermal fluctuations cause decoherence and destroy the interference pattern. This effect is visible in the upper panel of Fig. 7. Increase in the temperature of either of the reservoirs reduces the oscillations. However, this mechanism does not explain the results shown in the lower panel of Fig. 7 for γ=0.05, where the opposite tendency is clearly visible. Here, enhancement of the temperature of either of the reservoirs causes a significant

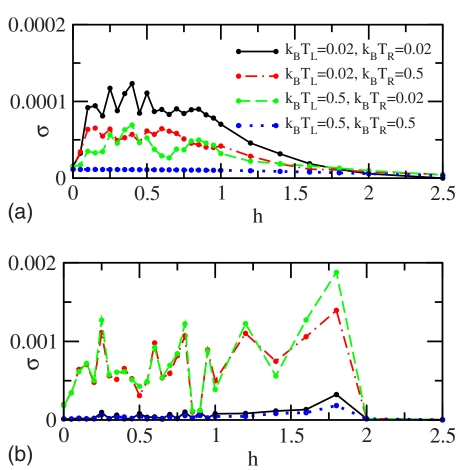

FIG. 7. (Color online) The standard deviation of spatial spin distribution σ, Eq. (28), as a function of the magnetic field h, for V=0 and h_i=0.25. The anisotropy parameters are γ=0 and γ=0.05 for upper and lower panels, respectively. The temperatures of left and right reservoirs are indicated in the legend. The points show our numerical results, whereas the lines are guidelines for the eyes.

increase in the long-range response. This nonequilibrium effect dominates over the destructive role of the thermal fluctuations.

Following Refs. 5 and 10 we expect for the thermal driving that the LRMC terminate at the critical magnetic field $h_c = 2|1-\gamma^2|$. Here, we consider $\gamma \ll 1$ and consequently expect the critical field to be $\simeq 2$. The analysis of Fig. 7 allows one to conclude that the spin distribution becomes nearly uniform (σ vanishes) when the magnetic field exceeds the value of 2. In the investigated regime $\gamma \ll 1$ this effect can be easily understood. Namely, for γ=0 and $|h| \geq 2$ the ground state of the XX model corresponds to fully polarized spins. Nevertheless, field dependencies of σ are very different in the isotropic XX and anisotropic XY models (see Fig. 7). In the former case, we observe a smooth gradual decrease in $\sigma(h)$ that starts already at $h \simeq 1$ while for $\gamma \neq 0$ there is a rather abrupt transition in the field $h_c \simeq 2$. In the case of the driving force V the above reasoning should be modified because the energy levels of left and right reservoirs are shifted by $h_L = -V/2$ and $h_R = V/2$, respectively. Since $-h_L$ and $-h_R$ play the role of the chemical potentials of the reservoirs [see Eq. (5)], full polarization of spins in the XY chain ($\gamma \ll 1$) is possible when either the top of the band $h+2$ is below $-V/2$ or the bottom of the band $h-2$ is above $V/2$. This mechanism explains the dependence $\sigma = \sigma(V)$ shown in Fig. 5 for h=2.5. While $\sigma \simeq 0$ for $V < 1$, it is small but nonzero for $V > 1$.

IV. DISCUSSION AND REMARKS

It is important to emphasize essential differences between the present approach and that in Refs. 5 and 10. We have applied the formalism of the nonequilibrium Green's functions, whereas the Lindblad,[5] or Redfield,[10] master equation has been used in Refs. 5 and 10. In the present case, the spectral properties of the reservoirs as well as their thermodynamic parameters (e.g., temperatures) explicitly enter the final equations, whereas in Refs. 5 and 10 all these quantities are encoded in two coupling constants. Therefore, in contradistinction to Refs. 5 and 10, we are able to test various physical mechanisms, which drive the system out of equilibrium. Our method of reasoning is also very different from that in Refs. 5 and 10. In the latter case the presence of LRMC has been directly established on the basis of the spin-spin correlators, whereas in the present approach LRMC show up as a strong enhancement of the long-range response to local perturbation. Despite the above differences both approaches lead to the same physical conclusion: in the anisotropic XY chain driven out of equilibrium there are LRMC provided the magnetic field does not exceed the critical value. The agreement between both methods strongly suggests that the nonequilibrium-induced LRMC are the *intrinsic* properties of the anisotropic XY model.

A separate problem concerns the possibility of experimental verification of the reported theoretical predictions. It remains an open question to what extent the anisotropic XY model is relevant to realistic quasi-one-dimensional spin systems. However, the fermionic Hamiltonian that arises after the Jordan-Winger transformation may describe also a one-dimensional triplet superconductor in a spin-polarized state. Therefore, one of the most promising candidates is a quasi-one-dimensional organic superconductor $(TMTSF)_2PF_6$. In this system superconductivity persists in the presence of magnetic field that is four times larger than the Pauli limit.[33,34] The Knight shift experiments provide another strong argument in favor of the triplet superconductivity.[35,36] Although application of a strong bias voltage (that in the present approach was denoted as the driving force *V*) would destroy superconductivity, the investigated effect sets on also in the presence of the heat current and should be visible as a nonequilibrium-driven charge order.

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